## Engineering Statistics IES 302

#### Dr. Prapun Suksompong prapun@siit.tu.ac.th Introduction



Office Hours: BKD 3601-7 Wednesday 15:30-16:30 Friday 9:30-10:30

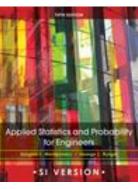
## **Course Organization**

• Course Website:

http://www2.siit.tu.ac.th/prapun/IES302/

- Lectures:
  - Wednesday 13:30-15:20 BKD 3507
  - Friday 10:40-12:30 BKD 3507
- Textbook:
  - Applied Statistics and Probability for Engineers
    - By Douglas C. Montgomery and George C. Runger
    - 5th Edition
    - There is a Student Companion Site.

STUDENT COMPANIO Montgomery: Applied			
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Browse by Chapter Select a Chapter Browse by Resource Data Sets	Anne Anne Anne Anne Anne Anne Anne Anne	Student Companion Site Montgomery: Applied Statistics and Probability for Engineers, 5th Edition International Student Version	Toolbox <ul> <li>How to use this site</li> <li>Table of contents</li> <li>Site map</li> </ul> Press ctrl-D to bookmark
Reading Quizzes			this page
<ul> <li>Answers to Selected Exercises</li> </ul>	Welcome		





## **Getting Info About This Course**

- The **syllabus** contains tentative information.
- I will announce **in class** and on the **web site** if there is any change.
- You are **responsible** for making sure that you obtain this information.
- Come to classes **on time** and listen carefully for **announcement**.

## Course Web Site

- Please check the course website regularly.
- Announcements
- References
- Handouts/Slides
- Calendar
  - Exams
  - HW due dates

www2.siit.tu.ac.th/prapun/IES302/

#### IES302: Engineering Statistics

#### Synopete

This course introduces the principles of probability and statistics to undergraduate students. The first part will focus on probability concepts including fundamentals of probability, events, conditional probability, discrets and course will discrete any control of the probability density function. The second part of the course will focus on applies statistics. The topics to be covered include sampling distributions, hypothesis testing, and regression analysis.

Announcements

+ Welcome to ISSE02: Real free to look around this site. (Posted # SPM on Jan 5)

#### General Information

Imstructor: Dr. Prepur Suksompong (prepur@sit.tu.ac.th)
 Office: SkS4801-9

• Office Hour: TEA

Course Syllabes

#### Class Information

 Textbook: Douglas C. Hontgomery and George C. Runger, <u>Applied Statistics and Probability for</u> Stighteers, 5 edition, June 2010

#### + Student Companion Site

#### · Enferences

- Douglas C. Nontgomery, George C. Runger, and Norma Farts Hubele, Engineering Statistics, Fifth Edition, Wiley, 2011
- A first course in probability / Sheldon Ross. Call No. QASTE RSE 2002
   Wild' course in probability / Sheldon Ross. Call No. QASTE RSE 1976
- Random signals for engineers using MATLAB and Mathcad / Richard C. Jaffe. Call No. 11(9)(02.9 )04 2000
- Probability theory and its applications, an introduction to / William Failer Call No. QA272 F27 Y288

#### · Free textbooks

- + Introduction to Probability by Charles M. Crinatead and J. Laurie Snell
- Leonard Modinow. The Drunkard's Halk: How Rendomness Rules Cur Lives. Pantheon; 5th Printing edition, 2008.
- Peter Olofsson. Probabilities: The Little Numbers That Rule Cur Lives. Wley, 2006.
- Hank Tijma, Understanding Probability: Chance Rules in Everyday Life. Cambridge University Press, 2 edition, August 2007.
- MATLAR Primer, Sth edition T. A. Davis, CRC Press, 2010.

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#### Course Website: Notes & Slides

- **PDF Notes** will be posted *before* the corresponding lectures.
  - Hard copies can also be purchased from the **copy center**.
- In lectures...
  - PDF Notes will be highlighted and updated with examples / comments.
  - Some lectures may use slides.
- The slides and updated notes will be **posted** *after* the corresponding lectures.
- I also frequently use Microsoft **OneNote** on my tablet instead of the whiteboard. The files will be exported as pdf and posted *after* the corresponding lectures.
- **Remind** me the day after the lecture if the notes/slides from the day before are still not posted on the web.

## **Course Outline**

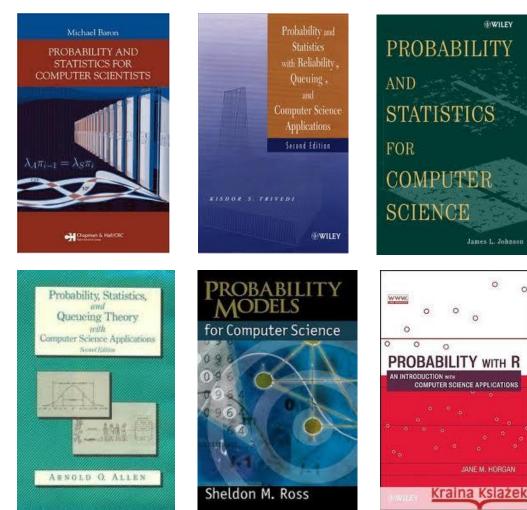
- Probability
- Random variables
- Discrete random variables
- Multiple random variables
- Continuous random variables
- **MIDTERM**: 24 Feb 2012 TIME 09:00 12:00
- Descriptive Statistics
- Random Sampling
- Sampling Distributions
- Confidence Interval
- Tests of Hypotheses
- Simple Linear Regression
- **FINAL**: 10 Apr 2012 TIME 13:30 16:30

#### Probability



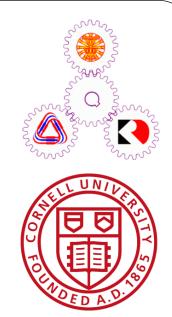


#### More Textbooks (for CS)



#### Me?

- Ph.D. from **Cornell** University, USA
- In Electrical and Computer Engineering
- Minor: Mathematics (Probability Theory)
- Ph.D. Research: Neuro-Information Theory
  - Modeling and analyzing neurons in human brain from communication engineering perspective.
- Current Research: Wireless Communication
  - Mobile Communications, WiFi (802.11)
- Best Teaching Award, 2009, SIIT











"I would found an institution where any person could find instruction in any study." Ezra Cornell On the Ithaca campus alone nearly 20,000 students (13,600 undergrad + 6,000 grad) representing every state and 120 countries choose from among 4,000 courses in 11 undergraduate, graduate, and professional schools.

Read Print Print Print

## Engineering Statistics IES 302

## Dr. Prapun Suksompong

prapun@siit.tu.ac.th

#### Part I: Probability

Office Hours: BKD 3601-7 Wednesday 15:30-16:30 Friday 9:30-10:30 "Les questions les plus importantes de la vie ne sont en effet, pour la plupart, que des problèmes de probabilité."

## "The **most important questions** of life are, for the most part, really only problems of **probability**."

#### THÉORIE

ANALYTIQUE

#### **DES PROBABILITES;**

#### PAR M. LE COMTE LAPLACE,

Chanciller du Sélexi-Conservatour, Grank-Olficier de la Légica Honosour; Meniller de l'Institut impérial et da Bureau des Longitudes de Prunes; des Sociétér systels de Londres et de Geningen; des Académies des Beinnes de Bustele, de Ducemarch; de Studie, de Prune, de Hollande, fitalie, etc.

Pierre Simon Laplace (1749 - 1827)

PARIS, M\*\* V\* COURCEER, fraprimers-Libraire pour les Mathématiques, quai des Angantias, a\* 57. 1812. "On voit, par cet Essai, que la théorie des probabilités n'est, au fond, que le bon sens réduit au calcul; elle fait apprécier avec exactitude ce que les esprits justes sentent par une sorte d'instinct, sans qu'ils puissent souvent s'en rendre compte."

## "One sees, from this Essay, that the theory of probabilities is basically just common sense reduced to calculus; it enables us to appreciate with exactness that which accurate minds feel with a sort of instinct, often without being able to account for it."

#### THÉORIE

ANALYTIQUE

#### DES PROBABILITES;

#### PAR M. LE COMTE LAPLACE,

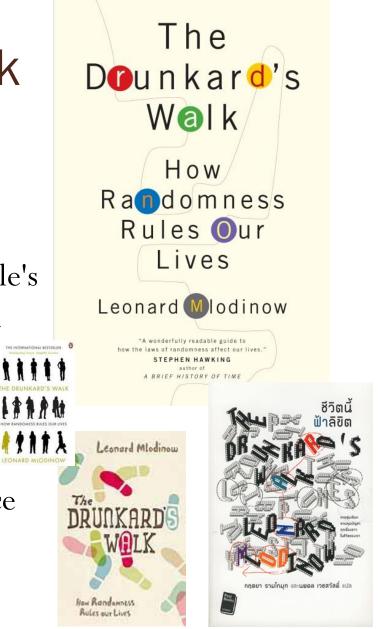
Chancelier du Stérat Conservatour, Grand-Officier de la Légiou d'Honnour; Metalere de l'Institut impérial et du Buranu des Longitables de Frances; des Sociétér systels de Locafere et de Ostimpor: des Académies des Beinness de Bussle, de Dauennarch, de Studie, de Prusse, de Hollande, B'Italés, etc.

Pierre Simon Laplace (1749 - 1827)

PARIS, M<sup>\*\*</sup> V<sup>\*</sup> COURCER, Imprimers-Libraire poor les Methémetiques, quis des Angasties, 2° 57. 1812.

### The Drunkard's Walk

- The Drunkard's Walk: How Randomness Rules Our Lives
- By Leonard Mlodinow
- Deals with randomness and people's inability to take it into account in their daily lives.
- A bestseller, and a "NY Times notable book of the year"
- Named "one of the 10 best science books of 2008" on Amazon.com.



[Thai Translation: ชีวิตนี้ ฟ้าลิขิต: การสุ่มเลือก ควบคุมบัญชา ทุกเรื่องราวในชีวิตของเรา]

## Leonard Mlodinow



- Euclid's Window: the Story of Geometry from Parallel Lines to Hyperspace
- Feynman's Rainbow: a Search for Beauty in Physics and in Life
- A Briefer History of Time
  - with Stephen Hawking
  - an international best-seller that has appeared in 25 languages.
- The Drunkard's Walk: How Randomness Rules our Lives
- Apart from books on popular science, he also has been a screenwriter for television series, including Star Trek: The Next Generation and MacGyver.

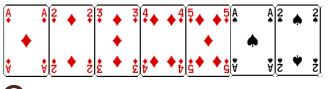
#### Watch Mlodinow's talk

- Delivered to Google employees
- About his book ("The Drunkard's Walk")



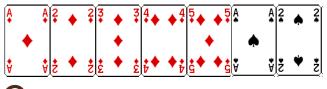
http://www.youtube.com/watch?v=F0sLuRsu1Do





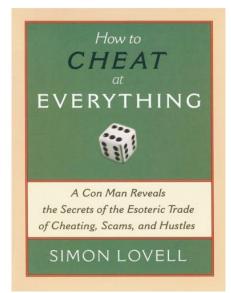
## The Seven Card Hustle

- Take five red cards and two black cards from a pack.
- Ask your friend to shuffle them and then, without looking at the faces, lay them out in a row.
- Bet that they can't turn over three red cards.
- Explain how the bet is in their favor.
  - The first draw is 5 to 2 (five red cards and two black cards) in their favor.
  - The second draw is 4 to 2 (or 2 to 1 if you like) because there will be four red cards and two black cards left.
  - The last draw is still in their favor by 3 to 2 (three reds and two blacks).
- The game seems heavily in their favor, butYOU, are willing to offer them **even money** that they can't do it!



#### The Seven Card Hustle

- Take five red cards and two black cards from a pack.
- Ask your friend to shuffle them and then, without looking at the faces, lay them out in a row.
- Bet that they can't turn over three red cards.
- Explain how the bet is in their favor.
- The game seems heavily in their favor, but YOU, are willing to offer them even money that they can't do it!



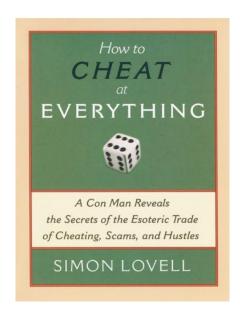
#### The Seven Card Hustle: Sol

The correct probability that they can do it is

$$\frac{\cancel{5} \times \cancel{4} \times \cancel{3}}{7 \times \cancel{6} \times \cancel{5}} = \frac{2}{7}$$
Alternatively, 
$$\frac{\binom{5}{3}}{\binom{7}{3}} = \frac{5!}{\cancel{3}!2!} \times \frac{\cancel{3}!4!}{7!}$$

$$= 5 \times 4 \times 3 \times \frac{1}{7 \times 6 \times 5}$$

$$= \frac{2}{7}$$



[Lovell, 2006]

### Monty Hall Problem (MHP): Origin

• Problem, paradox, illusion

20

- Loosely based on the American television game show
   Let's Make a Deal. (Thai CH7 version: ประตูดวง.)
- The name comes from the show's original host, **Monty Hall**.
- One of the most interesting mathematical brain teasers of recent times.





## Monty Hall Problem: Math Version

- Originally posed in a letter by Steve Selvin to the American Statistician in 1975.
- A well-known statement of the problem was published in Marilyn vos Savant's "Ask Marilyn" column in Parade magazine in 1990: "Suppose you're on a game show, and



"Suppose you're on a game show, and you're given the choice of three doors: Behind one door is a car; behind the others, goats. You pick a door, say No. 1, and the host, who knows what's behind the doors, opens another door, say No. 3, which has a goat. He then says to you, "Do you want to pick door No. 2?" Is it to your advantage to switch your choice?"

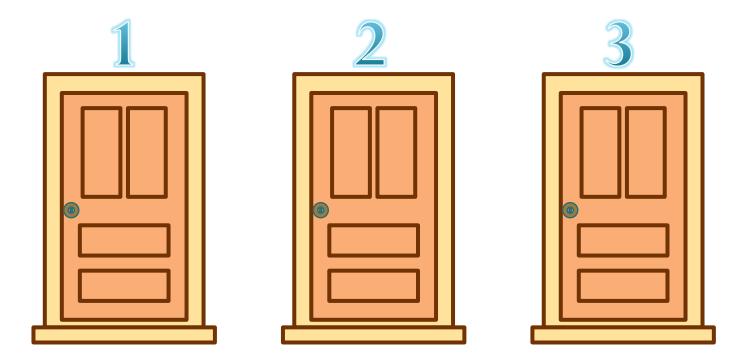


## Marilyn vos Savant

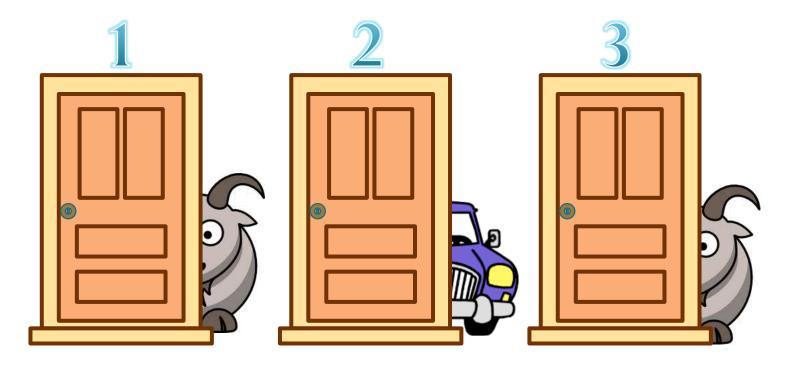
- Vos Savant was listed in each edition of the Guinness Book of World Records from 1986 to 1989 as having the "Highest IQ."
- Since 1986 she has written "Ask Marilyn"
  - Sunday column in Parade magazine
  - Solve puzzles and answer questions from readers



- There are three closed doors.
- They look identical.

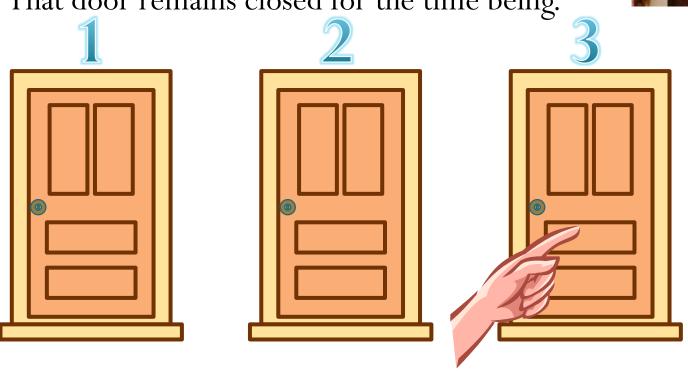


- Behind one of the doors is the star prize a car.
  - The car is initially equally likely to be behind each door.
- Behind each of the other two doors is just a goat.

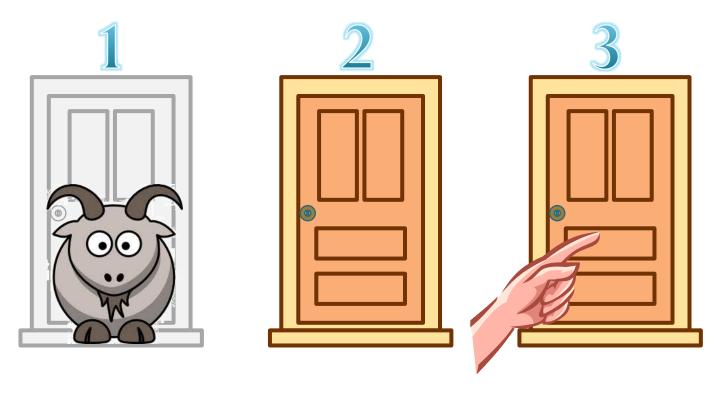


- Obviously we want to win the car, but do not know which door conceals the car.
- We are asked to choose a door.
- That door remains closed for the time being.

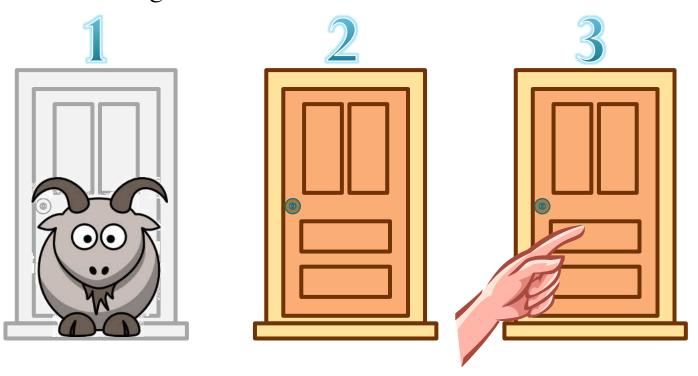




- The host of the show (Monty Hall), who knows what is behind the doors, now opens a door different from our initial choice.
  - He carefully picks the door that conceals a goat.
  - We stipulate that if Monty has a choice of doors to open, then he chooses randomly from among his options.



- Monty now gives us the options of either
  - 1. sticking with our original choice or
  - 2. switching to the one other unopened door.
- After making our decision, we win whatever is behind our door.





#### Monty Hall Problem

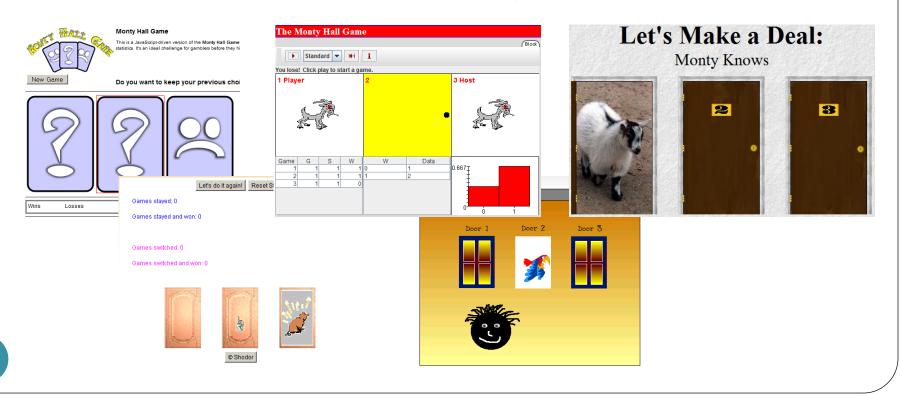
Assuming that our goal is to **maximize** our **chances** of **win**ning the car, what decision should we make?

- Will you do better by sticking with your first choice, or by switching to the other remaining door?
- Make no difference?

## Let's play!

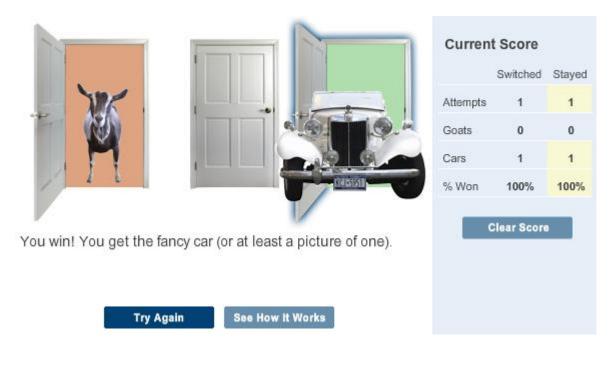
## **Interactive Monty Hall**

- <u>http://montyhallgame.shawnolson.net/</u>
- <u>http://www.shodor.org/interactivate/activities/SimpleMontyHall/</u>
- <a href="http://www.math.uah.edu/stat/applets/MontyHallGame.xhtml">http://www.math.uah.edu/stat/applets/MontyHallGame.xhtml</a>
- <u>http://scratch.mit.edu/projects/nadja/484178</u>
- http://www.math.ucsd.edu/~crypto/Monty/monty.html



#### **Interactive Monty Hall**

#### The New York Times's Version



http://www.nytimes.com/2008/04/08/science/08monty.html

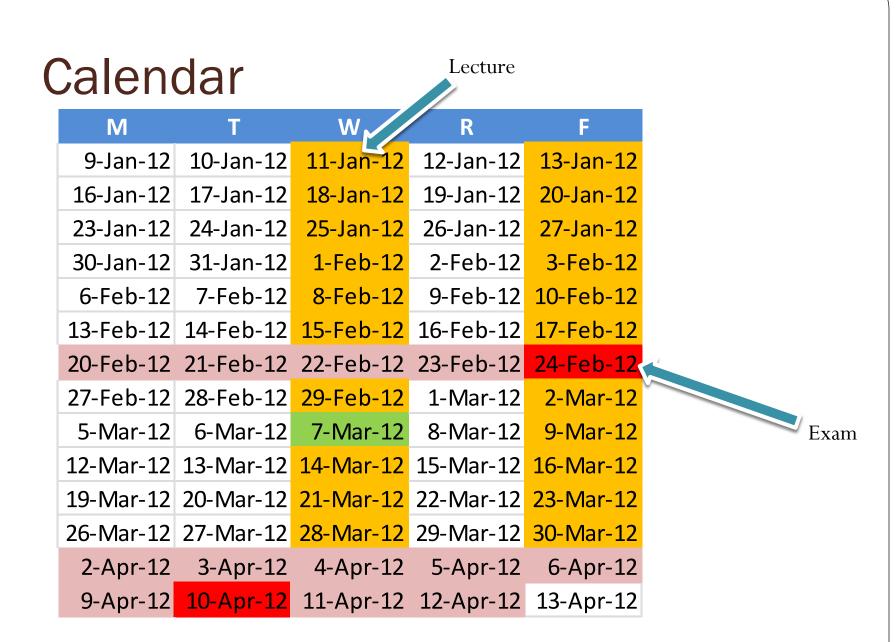
# Back to the boring administrative stuff!

## **Grading System**

• Coursework will be weighted as follows:

Assignments	5%
Class Participation and Quizzes	15%
Midterm Examination •24 Feb 2012 TIME 09:00 - 12:00	40%
Final Examination (comprehensive) •10 Apr 2012 TIME 13:30 - 16:30	40%

- Mark your calendars now!
- Late HW submission will be rejected.



Please Double-Check Exam Dates!

#### Calendar (Google)

Today	January	2012 👻			Week	Month	Agenda
Mon	Tue	Wed	Thu	Fri	Sat		Sun
26	27	28	29	30		31	Jan 1
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			Registration P				
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23	24	25	26	27		28	29
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Events shown in time	e zone: Bangk	ok			6	+ Goog	<b>e</b> Calendar

#### IES302: Engineering Statistics

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	Synappie					
1	This course hitroduces the principles of probability and statistics to undergraduate students. The first part will focus on probability concepts including fundamentals of probability, works, conditional probability, discrimes and concepts and the statistical probability devices from the second part of the sources will focus on applied statistics. The topics to be covered include sampling distributions, hypothesis testing, and regression analysis.					
	Announcements					
	<ul> <li>Welcome to IES202: Feel free to look around this site. (Posted @ SPM on [an 5)</li> </ul>					
	General Information					
	Imstructor: Dr. Prepun Eukaompong (prepunfalkturatin)     Office: aXD28001-9					
	Office Hour: T2A					
8	Course Syllabez     Class Information					
	Textibook: Dougle: C. Nontportery and Deorge C. Runger, Applied Statistics and Probability for Explorer, 5 etition, june 2010     State Comparison State     Beforements					
	Ecologias C. Montgomeny, George C. Runger, and Norma Faria Hubele, Engineering Statistics,					
	Fifth Edition, Wiley, 2011					
	<ul> <li>A first course is probability   Sheldon Ross. Cal No. Q4274 838 2002</li> <li>A first course in probability   Sheldon Ross. Cal No. Q4274 838 1076</li> <li>Random represent and explosers using INTLAS are limited   Richard C. Jaffe. Cal No. 1021022 [Q42 2005]</li> </ul>					
5	1K9Y02/9194 2000 • Probability theory and its applications, an introduction to / William Failer, Call No. Q4276 F87 1986					
	- Free textbooks					
	<ul> <li>Introduction to Probability by Charles M. Crinabead and J. Laurie Snell</li> <li>Leonard Modinou. The Distributive Mail: New Sendomness Sules Car Lives. Pantheory Sth</li> </ul>					
	Printing edition, 2008.					
	<ul> <li>Peter Giorason, Processiones: The Little Numbers That You Con Lives, whey, 2000.</li> <li>Henk Tijms, Understanding Probability: Chance Rules in Everyday Life. Cambridge University</li> </ul>					
	Press, 2 edition, August 2007.					
2	<ul> <li>MATLAS Primer, Sch edition T. A. Davis. GRC Press, 2010.</li> <li>Handours and Course Waterial</li> </ul>					
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	Problem Sec					
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	b. HW 2 (Due: )					
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#### **Class Participation**

- NOT the same as class attendance!
- If you come only to **receive**, you will fall **asleep**.
  - Do not simply sit quietly in the class.
- Need **interaction** between lecturer and students.
- Ask question when there is something that you don't understand.
  - Don't be shy!
  - It is very likely that your friends don't understand it as well.
- If you already understand what I'm presenting, SHOW ME!
  - Point out the errors/typos.
  - I will raise many issues/questions in class. Try to comment on them.

#### **Class Participation (2)**

- Record what you have done.
  - Submitted before the midterm and before the final.



Sirindhorn International Institute of Technology Thammasat University at Rangsit School of Information, Computer and Communication Technology

ECS 455: Self-Evaluation

#### Instructions

- The class participation score for this class is judged from how much you actively participate in the class discussion both inside and outside of the classroom.
- 2. Please honestly answer the following questions. Please provide as much information as possible.
- 3. Your answer will be read in detail and it may influence the actual score.

#### Quesions

- How many times have you been absent from the class? Are there any specific reason(s)? Please explain.
- How many times have you been late (> 3 mins) for the class? Are there any specific reason(s)? Please explain.
- How many times have you left the class early (> 3 mins)? Are there any specific reason(s)? Please explain.
- 4. How many times have you participated (provided comments, asked questions, answered questions, etc) in the lectures? Be specific. Provide some short description for each event. (You may put this on another sheet of paper.)

17/06/2010 : I asked question for the example of current source [Op-amp].
24/06/2010 : I answered for the Example 2.2.7. that's 111 2 is and series with site
But it's wrong. The correct answer is there's no series in that circuit.
01/07/2010 : I asked question " Is participate include with ask question after class".
Answer: Jes
08/07/2010 : I asked about Linear equation Why fix = 3x+1 is not a linear equation
eventually it is $y = mx + c$
Answer: Because it's not satisfy S(x)+koxx and S(x+x2) - S(x,)+S(x2)
15/07/2010 : I asked teacher to give an example of supermesh.
Answer: "We can use only supernode to solve the problems in this class.
You can find more in the textbook for supermesh".
20/07/2010 : I told teacher that we don't have class in static today so we can
move this class instead.
I asked :"Is the integrator and differtiator op-amp circuit is the same as in
the calculator function"" No for the op-amp it's amalog and for
calculator is digital.".
10/02/02/2 . I assert d sint = co>0, d co>0 = - sint
I corrected de the unit of voltage from A -> V.
I corrected the ont of voltage tout.

# Policy

- We will start the class on time and will finish on time.
  - Raise your hand and tell me immediately if I go over the time limit.
  - Does NOT mean that I will leave the room immediately after lecture.
    - I will stay and answer questions.



- Mobile phones *must* be turned off or set in silent mode.
- We may have some **pop quiz**zes (without prior warning or announcement) and in-class activities.
- Attendance and pop quizzes will be taken/given irregularly and randomly.
- Cheating will not be tolerated.

# Policy (con't)

- Feel free to stop me when I talk too fast or too slow.
- I will surely make some **mistakes** in lectures / HWs / exams.
  - Some amount of class participation scores will be reserved to reward the **first** student who inform me about each of these mistakes.
- Points on quizzes/ exercises/ exams are generally based on your entire solution, not your final answer.
  - You can get full credit even when you have the wrong final answer.
  - You may get **zero** even when you write down a right answer without justification.

#### Help and Office Hours

- Get some help!
  - Do not wait until the final exam time or after the grade is out.
  - Right after lecture is always a good time to ask question.
- Office Hours (BKD-3601)
  - Time:
    - Wednesday: 15:30-16:30 (right after class)
    - Friday: 9:30-10:30 (right before class)
  - Appointment can be made.
  - Tutorial session can be arranged.
  - Feel free to come to my office and chat!
  - Don't be shy.

# Warning

- This class is **difficult**.
  - Keep up with the lectures.
  - Make sure that you understand the concepts presented in the lecture before you go home.
- I will evaluate your understanding of the course regularly through
  - In class problems/activities
  - Quizzes
  - Exams



# Tips

- Almost everything including what I have written on my tablet will be saved and posted on web soon after class.
- No need to take detailed lecture notes (if you don't want to).
  - Put all of your energy into understanding the material.
  - Of course, there is always someone (in the class) who will take good notes anyway and you can (potentially) borrow or make a copy of the notes from them.
- Have fun with the materials presented in class.

#### Remarks

- Get as much legitimate help as you can
- Participate actively in class and outside of class
  - Record what you have done.
- If you feel that the class is very easy, you might overlook something.
- If you feel that the class is very difficult, you are probably not the only one who feel that way.
  - Don't give up. Chat with me.
  - It takes me a long time to feel comfortable with these materials; yet, I still make mistakes.
- My notation can be different from the textbook.
  - Every notation has some advantages and disadvantages.

#### Monty Hall Problem: a first revisit

Assuming that our goal is to **maximize** our **chances** of **win**ning the car, what decision should we make?

- Will you do better by sticking with your first choice, or by switching to the other remaining door?
- Make no difference?

#### Monty Hall Problem: vos Savant's Answer

"You **double** your chances of **win**ning by **switch**ing doors."



#### Monty Hall Problem: Controversy

- Approximately **10,000 readers**,
  - including nearly 1,000 with PhDs
  - (many of them math professors),
  - wrote to the magazine
  - **claim**ing the published solution was **wrong**.
- "You blew it," wrote a mathematician from George Mason University.
- From Dickinson State University came this: "I am in shock that after being corrected by at least three mathematicians, you still do not see your mistake."

[Mlodinow, 2008, p 42-45]



# Controversy (2)

- From Georgetown: "How many irate mathematicians are needed to change your mind?"
- And someone from the U.S. Army Research Institute remarked, "If all those Ph.D.s are wrong the country would be in serious trouble."
- When told of this, Paul **Erdos**, one of the leading mathematicians of the 20th century, said, "That's **impossible**."
  - Then, when presented with a formal mathematical proof of the correct answer, he still didn't believe it and grew angry.

# Let's learn some concepts so that we can analyze interesting examples!

#### Part I.1 Section 1-4



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#### 1 Probability and You

Whether you like it or not, probabilities rule your life. If you have ever tried to make a living as a gambler, you are painfully aware of this, but even those of us with more mundane life stories are constantly affected by these little numbers.

**Example 1.1.** Some examples from daily life where probability calculations are involved are the determination of insurance premiums, the introduction of new medications on the market, opinion polls, weather forecasts, and DNA evidence in courts. Probabilities also rule who you are. Did daddy pass you the X or the Y chromosome? Did you inherit grandma's big nose?

Meanwhile, in everyday life, many of us use probabilities in our language and say things like "I'm 99% certain" or "There is a onein-a-million chance" or, when something unusual happens, ask the rhetorical question "What are the odds?". [15, p 1]

#### 1.1 Randomness